Complete Decentralized Method for On-Line Multi-Robot Trajectory Planning in Well-formed Infrastructures

Michal Čáp[1] Jiří Vokřínek[1] Alexander Kleiner[2]

[1]Agent Technology Center,
Department of Computer Science,
CTU in Prague

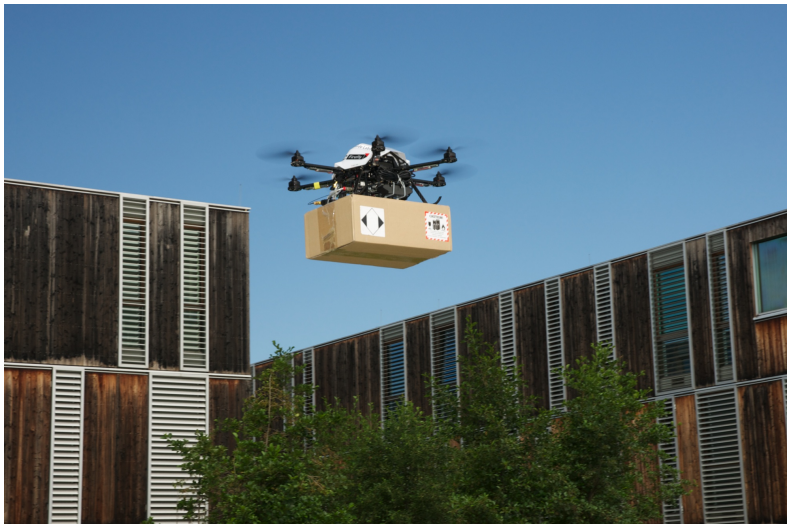
[2]iRobot Inc,
Pasadena, CA, USA

June 9, 2015

Motivation

(picture from http://raffaello.name)

Industrial intralogistics



(picture from http://bettstetter.com)

Automated transport UAVs

- Individual vehicles must not collide.

Motivation Scenario

Automated warehouse of an e-shop.

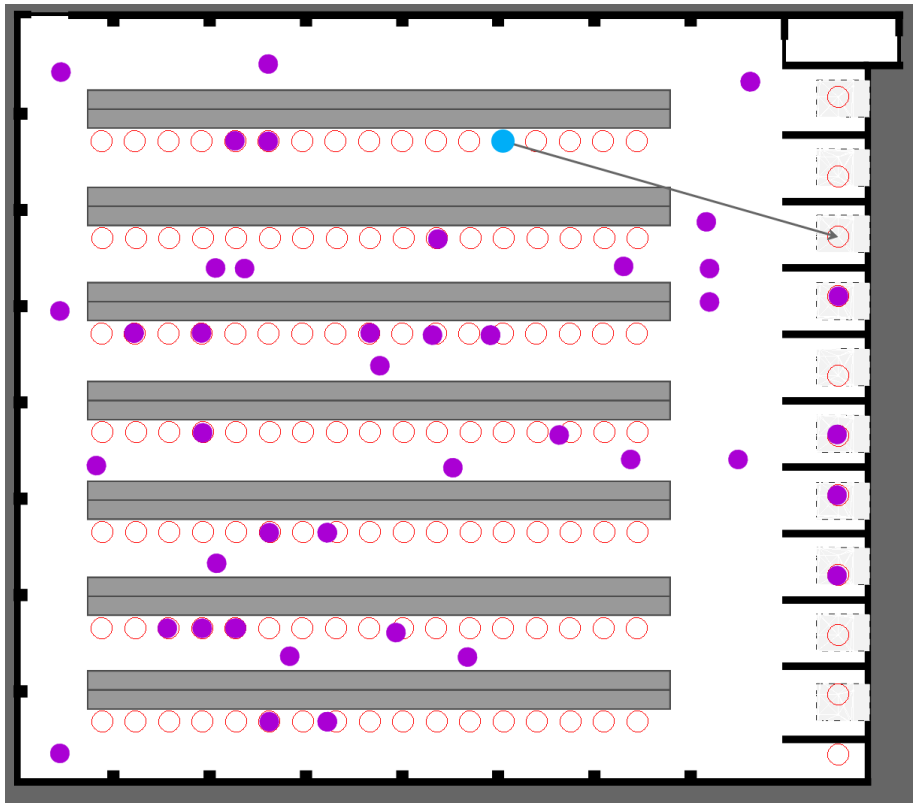
A product is ordered through a website:

1. A robot is sent from the depot to the shelf to get the product.
2. The robot carries the product to a human operator, who ships the product.

(picture from http://qrcodetracking.com)

Problem Definition

- static 2-d environment $\mathscr{W} \subseteq \mathbb{R}^2$
- finite set of *endpoints* $E \subset \mathscr{W}$
- occupied by n circular mobile robots with radius r and max. speed v
- *at any time*, a robot can be ordered to move from one endpoint to another (assigned a *relocation tasks*)
- **objective:** ensure that the destination of each relocation tasks will be reached without collision with other robots



Existing Techniques

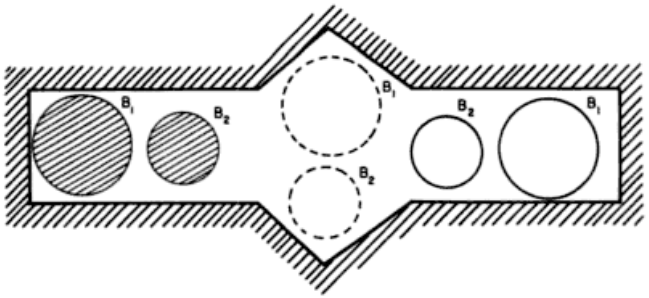
New relocation task $s \rightarrow g$ assigned to a robot:

- Reactive Approach
  - Follows shortest path from s to g (planned without considering other robots)
  - If potential collision detected – adjust immediate velocity to avoid the collision
  - Collision avoiding velocity computed using ORCA [Van Den Berg et al., 2011].
  - **Not guaranteed** – Collision solved locally, may lead to a deadlock
- Planning Approach
  - Interrupt the robots
  - Find coordinated trajectories from current position to destination of each robot
  - All robots start following the found coordinated trajectories
  - Dead-lock free

Complexity of Satisfying Multi-robot Path Planning

- Circles in a polygonal environment – **NP-hard** [Spirakis and Yap, 1984].

  All known complete algorithms are $O(c^n)$ [Ramanathan and Alagar, 1985, Wagner and Choset, 2015, Standley, 2010]



[Schwartz and Sharir, 1983]

Figure : Coordination of Disks

Our Contribution

We characterize a class of environments called *well-formed infrastructures*, where collision-free and dead-lock free trajectory for any relocation task can be computed in polynomial time.

COBRA

- Continuous Best-Response Approach (COBRA)
- Decentralized Approach
  - Each robots plans its trajectory using its on-board CPU
  - Uses token-based synchronization. The token carries current trajectories of all robots and can be held only by a single robot at a time.
- **Main idea:** When assigned a new relocation task, follow the optimal path to destination that avoids robots that were assigned their relocation tasks earlier.

COBRA – Algorithm

When-registered-at (p)

- $OtherTrajs \leftarrow$ **request-token**
- $Traj_i \leftarrow$ trajectory that stays at p forever
- add $Traj_i$ to $OtherTrajs$
- **release-token** $OtherTrajs$

Handle–relocation-task-to (g)

- $OtherTrajs \leftarrow$ **request-token**
- $Traj_i \leftarrow$ optimal trajectory to g avoiding $OtherTrajs$ [1]
- update the record of robot i in OtherTrajs with $Traj_i$
- **release-token** $OtherTrajs$
- **follow** $Traj_i$

[1] Requires a trajectory planner able to find an optimal trajectory for the robot amidst moving obstacles.

COBRA – Completeness

Assumptions:

- static environment
- perfect execution of trajectories
- non-interruptible relocation tasks

Theorem (completeness in well-formed infrastructures)

If COBRA is used to coordinate relocation tasks between endpoints of a well-formed infrastructure, then all relocation tasks will be carried out without collision.

Well-formed Infrastructure

Definition (infrastructure)

An infrastructure is a tuple $\langle \mathscr{W}, P \rangle$, where

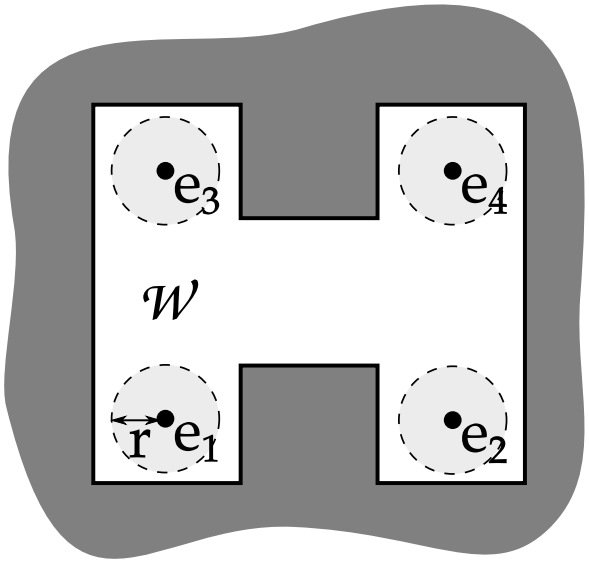
- $\mathscr{W} \subset \mathbb{R}^d$ is a set of obstacle-free positions (free space)
- $P \subset \mathscr{W}$ is a set of distinguished locations called endpoints

- robots move only between endpoints
- endpoints represent workplaces, parking places, etc.

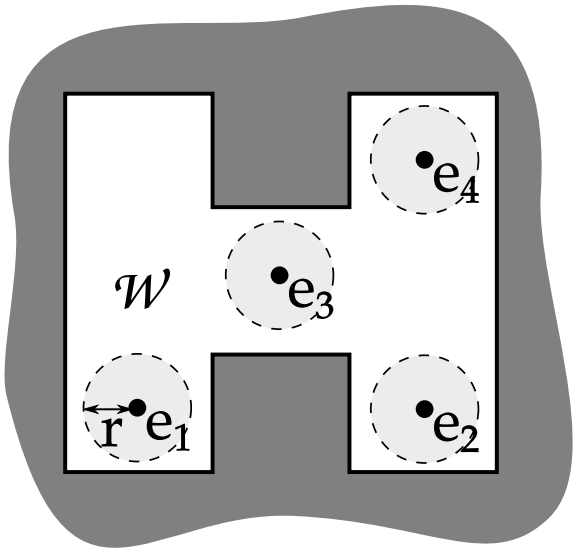
Definition (well-formed infrastructure)

An infrastructure $\langle \mathscr{W}, P \rangle$ is called **well-formed** for robots with max. radius r if there exists a path between any two endpoints p_1 and p_2 that avoids obstacles with r-clearance and all other endpoints with $2r$-clearance.

Well-formed Infrastructure – Example



Well-formed infrastructure



Ill-formed infrastructure

(There is no path between e_1 and e_4 that avoids e_3 with $2r$-clearance)

Well-formed Infrastructures in Real-world

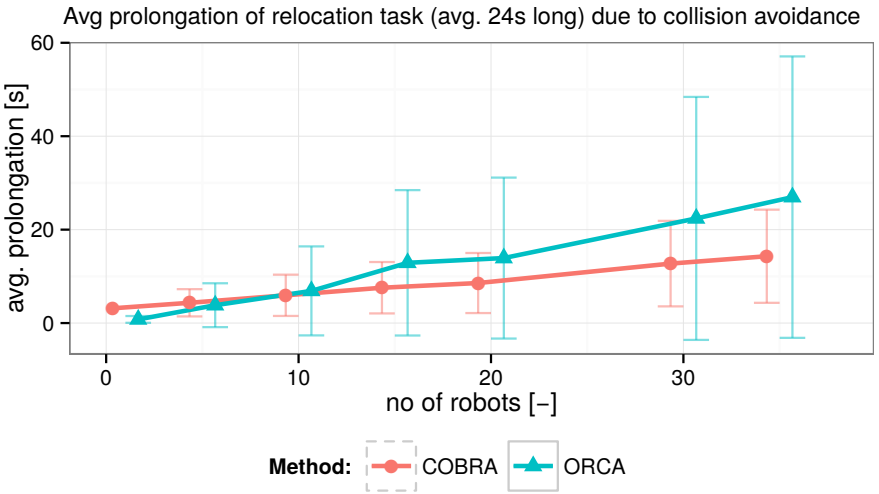
Human made environments are usually structured as well-formed infrastructures:

Complexity

Theorem (polynomial complexity)

The worst-case asymptotic complexity of a single relocation task handling using COBRA with time-extended roadmap planning is $O(n^2)$, where n is the number of robots in the system.

Results (Office Corridor)



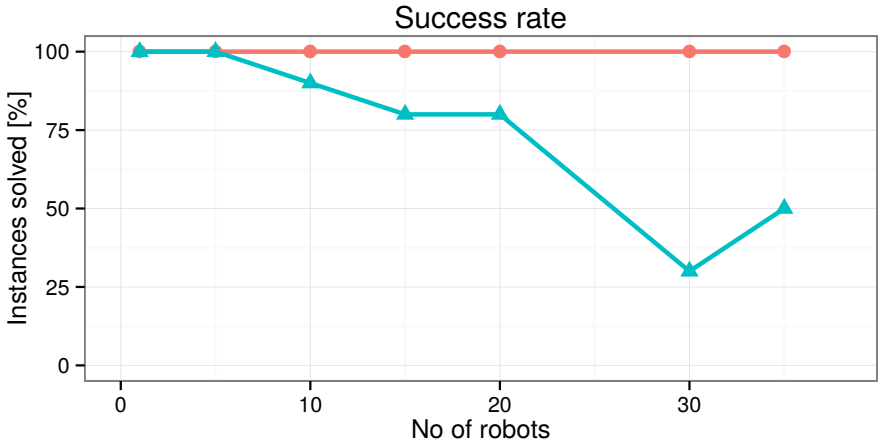
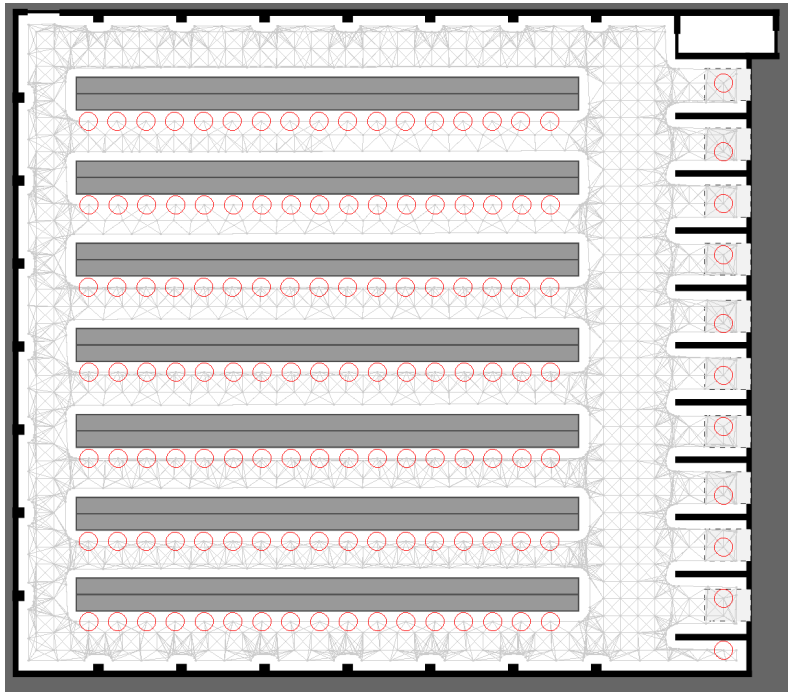


Illustration: COBRA in Office Corridor

(click to play)

Results (Warehouse)



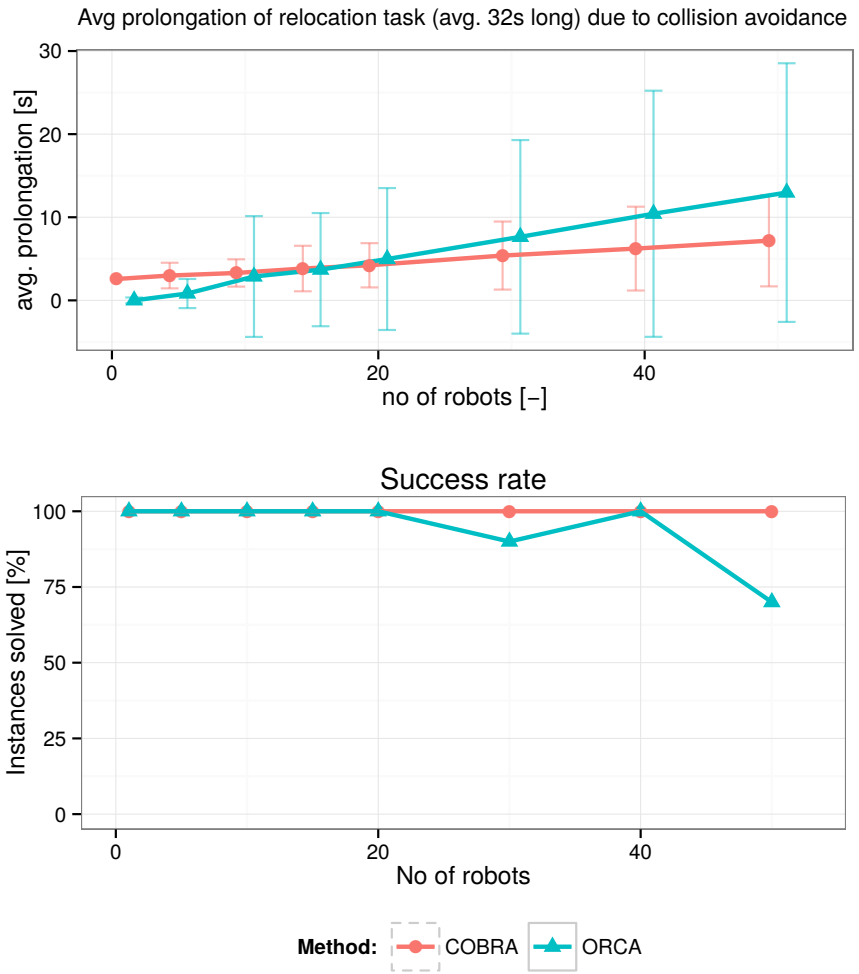


Illustration: COBRA in Warehouse

(click to play)

Conclusion

- Existing methods for collision avoidance in multi-robot systems are either
  - a) prone to deadlocks or
  - b) intractable.
- We characterized class of environments called **well-formed infrastructures** and designed and **polynomial guaranteed method COBRA** that can be used for trajectory coordination in such environments.

Benchmark instances and Java implementation available at:

http://agents.cz/~cap/
cap@agents.fel.cvut.cz

Questions?

References I

- Ramanathan, G. and Alagar, V. (1985).
  Algorithmic motion planning in robotics: Coordinated motion of several disks amidst polygonal obstacles.
  In *Robotics and Automation. Proceedings. 1985 IEEE International Conference on*, volume 2, pages 514–522.
- Schwartz, J. T. and Sharir, M. (1983).
  On the piano movers' problem: Iii. coordinating the motion of several independent bodies: the special case of circular bodies moving amidst polygonal barriers.
  The International Journal of Robotics Research, 2(3):46–75.
- Spirakis, P. G. and Yap, C.-K. (1984).
  Strong np-hardness of moving many discs.
  Inf. Process. Lett., 19(1):55–59.

References II

- Standley, T. S. (2010).
  Finding optimal solutions to cooperative pathfinding problems.
  In Fox, M. and Poole, D., editors, *Proceedings of the Twenty-Fourth AAAI Conference on Artificial Intelligence (AAAI)*. AAAI Press.
- Van Den Berg, J., Guy, S., Lin, M., and Manocha, D. (2011).
  Reciprocal n-body collision avoidance.
  Robotics Research, pages 3–19.
- Wagner, G. and Choset, H. (2015).
  Subdimensional expansion for multirobot path planning.
  Artificial Intelligence, 219:1 – 24.